Week 8: Unemployment: Introduction

Generic Efficiency-Wage Models

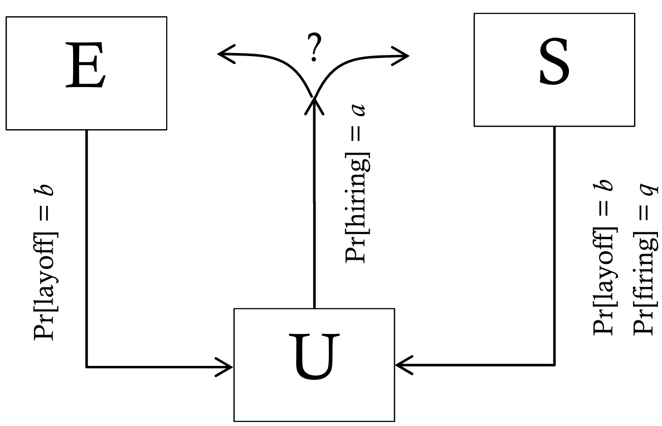
- Basic idea of efficiency wages: Raising a worker's wage makes her more productive
    - More effort
        - To keep job?
        - Why doesn't firm make effort a condition of employment and pay lower wages?
    - Improved applicant pool
    - Happier workers might be more productive
    - Higher wage might increase health (in developing countries)
- Basic models
    - Simple model of effort: $e = e(w)$
    - More complex model: $e = e(w, w_a, u)$
        - Firm must offer a higher wage than other firms (w_a) in order to get higher effort, for given level of unemployment rate
        - Could be simplified to $e = e(w - w_a, u)$
- Productivity effect
    - $Y = F(eL)$
    - $\Pi = F(eL) - wL$
    - Max Π:
        - $\frac{\partial \Pi}{\partial L} = eF'(eL) - w = 0$
        - $\frac{\partial \Pi}{\partial w} = LF'(eL)\frac{\partial e}{\partial w} - L = 0$
    - Solve together to get: $\frac{\partial e}{\partial w}\frac{w}{e} = 1$: Set wage at level where elasticity of effort with respect to wage is unity
    - Then $F'(eL) = \frac{w}{e}$ determines L
- Increase in MPL would lead to higher L but not higher w
    - This is strongly consistent with data in a way that the RBC model does not explain
    - However, if w_a or u changed, then the optimal wage would respond
    - This is one justification for the "wage adjustment equation" that Romer uses from time to time in place of assuming that the wage clears the labor market
- Can *all* firms pay an efficiency wage?
    - If all firms are symmetric, then all end up paying the same wage, so no individual firm pays a wage higher than others ($w = w_a$)

- But driving up wage leads to a persistent excess supply of labor and unemployment, which keeps workers working hard out of fear of becoming unemployed: "Unemployment as a worker-discipline device"

Shapiro-Stiglitz Model

Basic setup

- Shapiro-Stiglitz model tries to get inside the *e* function to model workers' decisions about how hard to work
  - Application of dynamic programming: mathematical technique that macroeconomics use a lot
- A worker can be in any of three states:
  - *E* means she is working hard
  - *S* means she is shirking
  - *U* means she is unemployed
- We analyze movement between states in continuous time
  - Hazard rate = instantaneous probability (per year) of moving into or out of a state



  - Employed worker chooses whether to be in *E* or *S*
  - q is the penalty for shirking in terms of a probability of getting caught and fired
    - In world of perfect monitoring of worker performance, firms can fire workers immediately and $q \to \infty$
    - If it is totally impossible for firms to monitor workers, then $q = 0$
- Instantaneous probabilities of moving:
  - Movement can happen at any moment, but probabilities are still expressed in "per period" rate
  - Intuition based on frequency of layoff opportunities:

        - Suppose the period is one year
        - If one can only be laid off at end of year, then probability of still being employed after a year is $(1-b)^1$
        - If one can be laid off at middle or end of year, then $\left(1-\frac{1}{2}b\right)^2$
        - If one can be laid off at end of any quarter: $\left(1-\frac{1}{4}b\right)^4$
        - If at end of any month: $\left(1-\frac{1}{12}b\right)^{12}$
        - If any day: $\left(1-\frac{1}{365}b\right)^{365}$
        - As opportunities for layoffs become continuous: $\lim_{n\to\infty}\left(1-\frac{1}{n}b\right)^n = e^{-b}$
    - Probability that someone starting in *E* is still in *E* after Δt is $e^{-b\Delta t}$
    - Probability that someone starting in *S* is still in *S* after Δt is $e^{-(b+q)\Delta t}$
    - Probability that someone starting in *U* is still in *U* after Δt is $e^{-a\Delta t}$
- Working utility: $U = \int_0^\infty e^{-\rho t} u(t)\,dt$, with $u(t) = w(t) - \bar{e}$ if employed and working, $u(t) = w(t)$ if employed and shirking, and $u(t) = 0$ if unemployed
- Firm's profit with *L*(*t*) working hard and *S*(*t*) shirking is $\Pi(t) = F\left(\bar{e}L(t)\right) - w(t)\left[L(t) + S(t)\right]$

Dynamic programming

- Fundamental underlying equation of dynamic programming is the Bellman equation, which relates to the lifetime expected utility of someone who is currently in state *i*:
  $$V_i(0) = \lim_{\Delta t\to 0} V_i(\Delta t) \equiv \lim_{\Delta t\to 0}\left\{\int_{t=0}^{\Delta t} u(t\,|\,\text{state at } 0 = i)\,dt + e^{-\rho\Delta t} E\left[V(\Delta t\,|\,\text{state at } 0 = i)\right]\right\}$$
- For state *E*, the Bellman equation is
  $$V_E(\Delta t) = \int_{t=0}^{\Delta t} e^{-\rho t}\left[e^{-bt}(w-\bar{e}) + \left(1-e^{-\rho t}\right)(0)\right]dt$$
  $$+e^{-\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t) + \left(1-e^{-b\Delta t}\right)V_U(\Delta t)\right]$$
    - Interpretation of expressions:
        - Integral is utility gained over *t* between time 0 and Δt
            - Bracketed sum is expected utility at *t* given probabilities of being employed and unemployed
                - Discount factor in front
                - e^{-bt} is probability that worker is still *E* at *t* given *E* at 0
                - $(w - \bar{e})$ is utility gained at each moment in state *E*
                - $\left(1-e^{-bt}\right)$ is probability of having been laid off before *t*
                - (0) is the utility obtained at *t* if unemployed (laid off)

- Discount factor in front of second bracketed term discounts for period 0 to Δt
  - Bracketed term is expected value of utility over rest of life given E at time 0:
    - $e^{-b\Delta t}$ is probability still employed at Δt
    - $V_E(\Delta t)$ is discounted rest-of-life value of utility at time Δt if still in state E
    - $\left(1-e^{-b\Delta t}\right)$ is probability that worker has moved to U by Δt
    - $V_U(\Delta t)$ is discounted rest-of-life value of utility at time Δt if in state U

- Evaluating the definite integral:

$$\int_{t=0}^{\Delta t} e^{-(\rho+b)t}\left(w-\bar{e}\right)dt = \left[-\frac{\left(w-\bar{e}\right)}{\rho+b}e^{-(\rho+b)\Delta t}\right]-\left[-\frac{\left(w-\bar{e}\right)}{\rho+b}e^{-(\rho+b)0}\right]$$

$$=\left[-\frac{w-\bar{e}}{\rho+b}e^{-(\rho+b)\Delta t}\right]+\frac{w-\bar{e}}{\rho+b}=\left(1-e^{-(\rho+b)\Delta t}\right)\frac{w-\bar{e}}{\rho+b}$$

- Substituting into Bellman equation:

$$V_E(\Delta t)=\frac{w-\bar{e}}{\rho+b}\left(1-e^{-(\rho+b)\Delta t}\right)+e^{\rho\Delta t}\left[e^{-b\Delta t}V_E(\Delta t)+\left(1-e^{-b\Delta t}\right)V_U(\Delta t)\right]$$

- Bringing the V_E terms to the left-hand side:

$$V_E(\Delta t)\left(1-e^{-(\rho+b)\Delta t}\right)=\frac{w-\bar{e}}{\rho+b}\left(1-e^{-(\rho+b)\Delta t}\right)+e^{\rho\Delta t}\left(1-e^{-b\Delta t}\right)V_U(\Delta t)$$

$$V_E(\Delta t)=\frac{w-\bar{e}}{\rho+b}+\frac{e^{-\rho\Delta t}\left(1-e^{-b\Delta t}\right)}{1-e^{-(\rho+b)\Delta t}}V_U(\Delta t)$$

- Taking the limit as $\Delta t \to 0$, both the numerator and denominator of the expression in front of V_U go to zero.
  - Applying L'Hôpital's Rule, we can show that

$$\lim_{\Delta t\to 0}\frac{e^{-\rho\Delta t}\left(1-e^{-b\Delta t}\right)}{1-e^{-(\rho+b)\Delta t}}=\lim_{\Delta t\to 0}\frac{-\rho e^{-\rho\Delta t}-\left(-\rho-b\right)e^{-(\rho+b)\Delta t}}{\left(\rho+b\right)e^{-(\rho+b)\Delta t}}=\frac{b}{\rho+b}$$

- Thus, $V_E=\dfrac{w-\bar{e}+bV_U}{\rho+b}$, or

$$\left(\rho+b\right)V_E=\left(w-\bar{e}\right)+bV_U$$

$$\rho V_E=\left(w-\bar{e}\right)+b\left(V_U-V_E\right)$$

- This last equation has a useful interpretation that we will apply to get the values of the other states without all the math:
  - The left-hand side is the "utility return on being in state E"
    - This is the discount rate ρ times that capital value of being in state E

        - Analogous to multiplying an interest rate (of return) times the capital value of an asset to get an annual flow of returns
    - The first term on the right is the "dividend" earned while in state E
        - Each instant that the individual is in E he or she gets $w - \bar{e}$
    - The last term on the right is the "expected capital gain" from being in state E
        - Probability of changing state is b
        - Change in capital value if state is changed is $V_U - V_E < 0$
        - Expected change in value is the product of the probability of changing state times the change in value if you do change state
- Can apply the "utility return" method to get V_S and V_U (or you can do the lengthy derivation if you want):
    - $\rho V_S = w + (b + q)(V_U - V_S)$
    - $\rho V_U = 0 + a(V_E - V_U)$, assuming that the individual works rather than shirks with hired.
        - (It doesn't matter, because we are going to set $V_E = V_S$ as a condition for equilibrium anyway.)
- Summarizing the key relationships:

$$\rho V_E = (w - \bar{e}) + b(V_U - V_E)$$
$$\rho V_S = w + (b + q)(V_U - V_S)$$
$$\rho V_U = a(V_E - V_U)$$

Decision-making and equilibrium

- **No shirking**
    - Firm will always pay a wage high enough to keep workers from shirking, because if workers shirk then the firm incurs wage cost but gets no output
    - Assume that workers work if and only if $V_E \geq V_S$, in other words, they work if the values are equal
    - Setting $\rho V_E = \rho V_S$,

$$w - \bar{e} - b(V_E - V_U) = w - (b + q)(V_E - V_U)$$
$$V_E - V_U = \frac{\bar{e}}{q} > 0.$$

        - Firms set wage high enough that working is more desirable than being unemployed, so workers have something to lose if they are fired or laid off

- Solving for the wage from the ρV_E equation:

$$w = \bar{e} + \rho V_E + b(V_E - V_U)$$
$$= \bar{e} + (b+\rho)(V_E - V_U) + \rho V_U$$
$$= \bar{e} + (b+\rho+a)(V_E - V_U), \text{ because } \rho V_U = a(V_E - V_U)$$
$$w = \bar{e} + (a+b+\rho)\frac{\bar{e}}{q}.$$

  - Wage that firms must set to assure no shirking depends on disutility of working hard ($\bar{e}$), probability of being caught shirking (q), probability of being rehired if unemployed (a), and b and ρ.

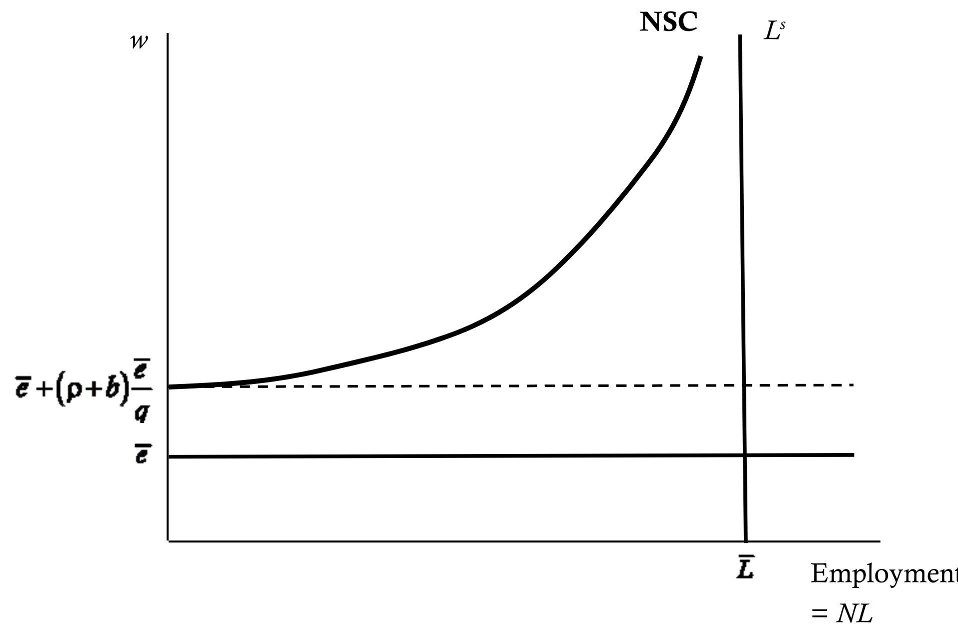
- **Equilibrium**
  - In steady state with constant unemployment rate, flow of workers from E to U must balance flow from U to E:
    - If there are N firms and each one hires L workers, then total employment is NL
    - Suppose that the total labor force is fixed at $\bar{L}$
    - Number unemployed is $\bar{L} - NL$
    - Balancing flows are $bNL = a(\bar{L} - NL)$, so

$$a = \frac{bNL}{\bar{L} - NL} \text{ and}$$
$$a + b = \frac{\bar{L}}{\bar{L} - NL} b = \frac{1}{u} b \text{, where } u \text{ is the unemployment rate}$$

  - Substituting into the no-shirking wage,

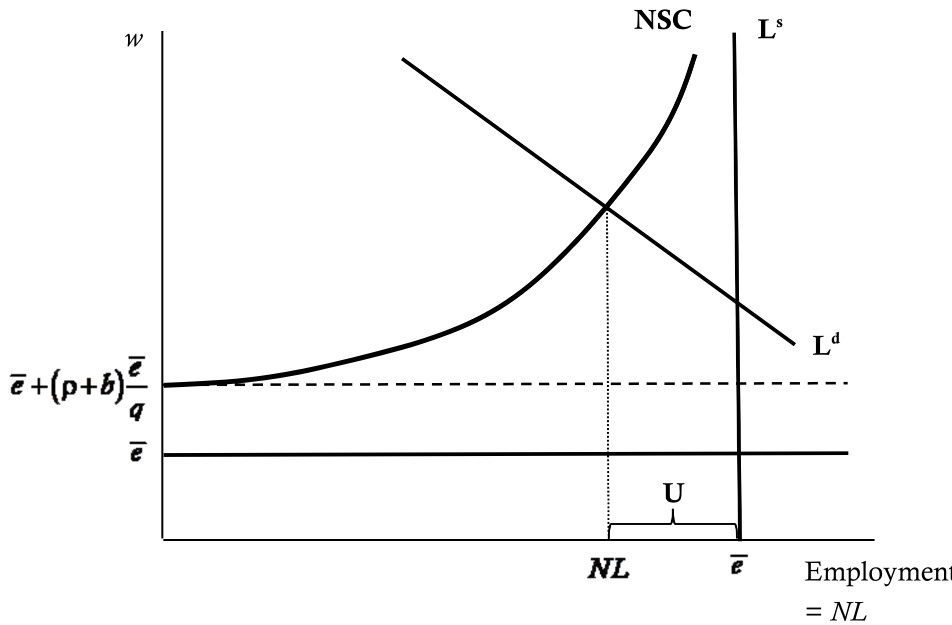
$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL} b\right)\frac{\bar{e}}{q}$$ is the **no-shirking condition**

    - Firms must pay a wage at least equal to this level in order to avoid shirking
    - Can be written as $w = \alpha + \beta\frac{1}{u}$, which is a rectangular hyperbola in the unemployment rate
    - Graphing w against NL gives:



- Effects of parameters on NSC:
  $\bar{e}\uparrow \Rightarrow$ NSC $\uparrow$
  $\bar{L}\uparrow \Rightarrow$ NSC $\rightarrow$
  $b\uparrow \Rightarrow$ NSC $\nwarrow$
  $q\uparrow \Rightarrow$ NSC $\downarrow$
- $q \to \infty$ means that shirkers get caught immediately and NSC becomes backward L at $\bar{e}$ and $\bar{L}$:
  - Workers all work if wage is greater than or equal to $\bar{e}$
- With finite q, the NSC is like a supply curve for labor, telling firms how much they must (collectively) pay in order to get a certain number of workers to work hard

- **Labor demand**
  - For individual firm, $\Pi = F[\bar{e}L] - wL$
  - Profit-maximization: $\frac{\partial \Pi}{\partial L} = \bar{e}F'[\bar{e}L] - w = 0$, given the w on the NSC
  - Labor-demand curve for each of N firms comes from $F'[\bar{e}L] = \frac{w}{\bar{e}}$, which is declining in L, so labor demand curve slopes downward as usual
    - Having to offer a higher efficiency wage means it is only profitable to hire a smaller number of workers



- If firms had perfect information about shirkers so $q = \infty$, then equilibrium occurs at full employment, where $L^d = L^s$
- With monitoring costs, equilibrium occurs where L^d = NSC and unemployment is the gap $\bar{L} - NL$
- Title of paper: "Unemployment as a worker discipline device"
- No firm pays higher wage than any other, so efficiency wage in aggregate means working hard because getting fired mean being unemployed (not going to a lower-wage firm)

Issues

- **Bonding**
    - How about having employee post a bond a hiring that is forfeited if he shirks?
    - This would allow firms to hire the entire labor force at the equilibrium wage (no unemployment)
    - Enforcement might be difficult: firm has incentive to claim shirking and seize bond, even if worker is not shirking
    - Workers might not be sufficiently liquid to pay up front
    - We see this to some extent in structure of labor compensation
        - Delayed vesting of retirement plans: Some worker benefits are not earned until worker has completed a certain number of years
        - Rising wage scale over time
            - More senior workers may not be more productive, but by offering higher wages to them it encourages workers to avoid firing (and quitting)
- **Costs of monitoring**

  - o One can imagine a model in which firms choose between paying an efficiency wage and incurring costs of monitoring more closely
  - o A decline in monitoring costs (due to better surveillance techniques, perhaps) would lower wage and increase employment
    - ▪ Could this help explain blue-collar wage stagnation since 1980s?

Search and Matching Model

Basic model setup

- Workers and jobs are heterogeneous
  - o Matching is a time-consuming process involving matching vacant job with unemployed worker
- Workers can either be employed/working or unemployed/searching:
  - o There is mass one of workers with fraction E employed and U unemployed: $E + U = 1$
- When a worker is employed, he or she produces output at constant flow rate y and earns a wage of $w(t)$
- When a worker is unemployed, he receives a benefit of $b > 0$ (either unemployment benefit payments or leisure utility, or both)
- Firms have a pool of jobs, some of which (F) are filled and some of which (V) are vacant
- A firm incurs a constant flow cost $c < y$ of maintaining a job, whether it is vacant or filled
  - o This is a simplification, but think about all of the overhead personnel costs of keeping track of employees and the search costs of hiring for a new one
  - o We just assume that they are the same (for simplicity)
  - o $\Pi(t) = y - w(t) - c$ for each filled job
    $\Pi(t) = -c$ for each vacant job
  - o Vacancies/jobs are costless to create (but expensive to maintain)
- Both workers and firms have a discount rate of r
- Matching function matches members of the pool of unemployed workers with members of the pool of vacant jobs:
  $M(t) = M\left[U(t), V(t)\right]$, with $M_U > 0$, $M_V > 0$
- Employment matches end (through retirement, firm contraction, etc.) at a constant rate λ, so
  $\dot{E}(t) = M\left[U(t), V(t)\right] - \lambda E(t)$

Matching function

- Matching function is like a production function, but it need not have constant returns to scale:
  - Thick-market effects may make it easier for workers/jobs to find one another if there are many out there: increasing returns to scale
  - Congestion effects might make it more difficult to find one another if job-search resources are congested: decreasing returns to scale
- We assume CRTS and Cobb-Douglas marching function:
  $M[U(t),V(t)] = k[U(t)]^{1-\gamma}[V(t)]^{\gamma}$, with k being an index of the efficiency of job search
  - The **job-finding rate** $a(t)$ (same as Shapiro-Stiglitz a) is the rate at which unemployed workers find jobs: $M[U(t),V(t)]/U(t)$
    - With CRTS:
      $$a(t) = m[\theta(t)], \text{ with } \theta(t) \equiv \frac{V(t)}{U(t)} \text{ and } m[\theta(t)] \equiv M(1,\theta(t))$$
    - θ is an indicator of labor market looseness: higher θ means more job vacancies or fewer unemployed workers, making it easier for workers to find jobs
    - With Cobb-Douglas: $a(t) = m[\theta(t)] = k\theta^{\gamma}$
  - The **job-filling rate** $\alpha(t)$ is the rate at which vacant jobs are filled: $M[U(t),V(t)]/V(t)$
    - With Cobb-Douglas: $\alpha(t) = \dfrac{m[\theta(t)]}{\theta(t)} = k\theta^{\gamma-1}$
- **Nash bargaining**
  - There is no "market wage" because each individual and job are unique
  - The wage is set to divide up the mutual gains from making the match, with share ϕ going to the worker and $(1-\phi)$ going to the firm
  - The value of ϕ will depend on institutions in the economy (and could depend on market conditions)

Decision-making

- Dynamic programming:
  - What is the value to worker of being in state E or in state U?
  - What is the value to firm of having filled job F or vacant job V?
  - Here, we consider the possibility that the economy may not always be in the steady state, so there can be a change in the value V_i over time, which adds (if positive) to the benefit of being in that state (like a capital gain)

- For the worker:

  $rV_E(t) = w(t) + \dot{V}_E(t) - \lambda\left[V_E(t) - V_U(t)\right]$

  $rV_U(t) = b + \dot{V}_U(t) + a(t)\left[V_E(t) - V_U(t)\right]$

- For the firm:

  $rV_F(t) = \left[y - w(t) - c\right] + \dot{V}_F(t) - \lambda\left[V_F(t) - V_V(t)\right]$

  $rV_V(t) = -c + \dot{V}_V(t) + \alpha(t)\left[V_F(t) - V_V(t)\right]$

Equilibrium conditions

- In the steady state, all of the $\dot{V}$ terms are zero, so we will now neglect them
- Also, in steady state, both a and α are constant
- **Evolution of number unemployed** is $\dot{E}(t) = \left[U(t)\right]^{1-\gamma}\left[V(t)\right]^{\gamma} - \lambda E(t)$ and must be zero in steady-state
- **Nash bargaining**:
  - Suppose that the total gain from match is X, of which worker gets ϕX and firm gets $(1-\phi)X$
  - $\begin{matrix}(V_E - V_U) = \phi X \\ (V_F - V_V) = (1-\phi)X\end{matrix}$, so $X = \dfrac{V_E(t) - V_U(t)}{\phi} = \dfrac{V_F(t) - V_V(t)}{1-\phi}$

    and $V_E(t) - V_U(t) = \dfrac{\phi}{1-\phi}\left[V_F(t) - V_V(t)\right]$
- **Vacancies are costless to create**: $V_V(t) = 0$

Solution

- Solve model in terms of E and V_V
- Subtracting V_U from V_E yields

  $r\left[V_E(t) - V_U(t)\right] = w(t) - b - \left(\lambda + a(t)\right)\left[V_E(t) - V_U(t)\right]$, or

  $V_E - V_U = \dfrac{w-b}{a+\lambda+r}$
- Doing the same to V_F and V_V gives

  $V_F - V_V = \dfrac{y-w}{\alpha+\lambda+r}$
- From the Nash bargaining condition:

  $\dfrac{w-b}{a+\lambda+r} = \dfrac{\phi}{1-\phi}\dfrac{y-w}{\alpha+\lambda+r}$,

  $w = b + \dfrac{(a+\lambda+r)\phi}{\phi a + (1-\phi)\alpha + \lambda + r}(y-b)$

- As a benchmark example, suppose that $b = 0$ (no unemployment benefits), $a = \alpha$ (job-finding rate = job-filling rate), and $\phi = ½$ (bargaining shares are equal)
  - In this case, $w = \frac{(a+\lambda+r)\frac{1}{2}}{a+\lambda+r} y = \frac{1}{2} y$.
  - Workers get half of their product and firms get half
- Higher ϕ means workers get higher wage
- Higher b means workers get higher wage
- Higher a or lower α means workers get higher wage

- Value of vacancy:

$$\begin{aligned} rV_V &= -c + \alpha\left[V_F - V_V\right] \\ &= -c + \alpha \frac{y - w}{\alpha + \lambda + r} \\ &= -c + \frac{(1-\phi)\alpha}{\phi a + (1-\phi)\alpha + \lambda + r}(y - b) \end{aligned}$$

- $\dot{E} = 0 \Rightarrow M\left[U, V\right] \equiv aU \equiv a(1-E) = \lambda E$, so

$a = \frac{\lambda E}{1-E}$, which is increasing in E

- $\lambda E = M(U,V) = kU^{1-\gamma}V^{\gamma} = k(1-E)^{1-\gamma}V^{\gamma}$, so

$$V = k^{-\frac{1}{\gamma}}(\lambda E)^{\frac{1}{\gamma}}(1-E)^{\frac{\gamma-1}{\gamma}}$$

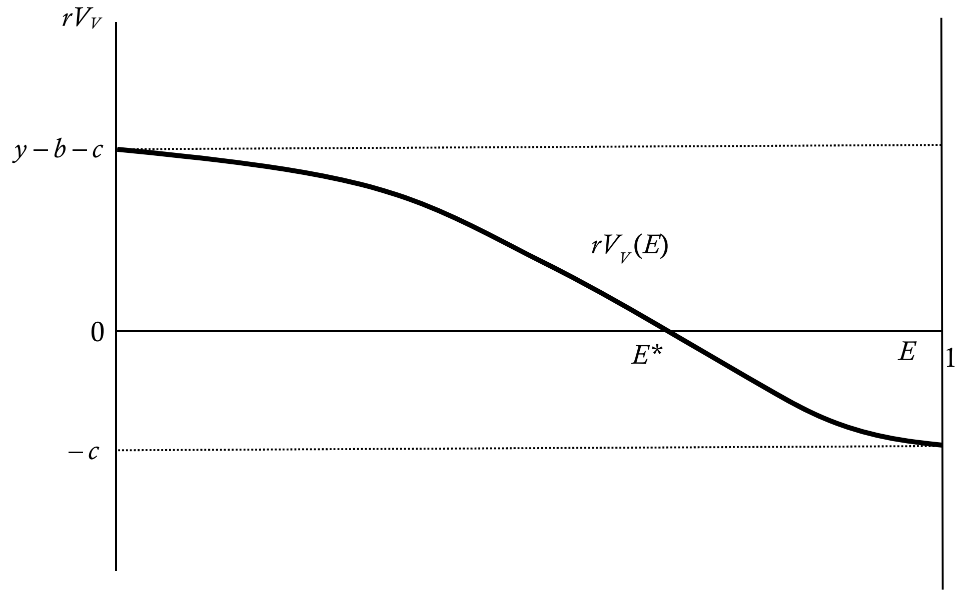
$$\alpha = M / V = k^{\frac{1}{\gamma}}(\lambda E)^{\frac{\gamma-1}{\gamma}}(1-E)^{\frac{1-\gamma}{\gamma}}$$

α is decreasing in E because $\gamma < 1$

- Free creation of vacancies implies that $V_V = 0$ in steady state, so

$$rV_V = -c + \frac{(1-\phi)\alpha(E)}{\phi a(E) + (1-\phi)\alpha(E) + \lambda + r}(y - b) = 0$$

- When $E = 1$, $\alpha = 0$ (it takes forever to fill a vacancy because there are no unemployed workers)
  - $rV_V = -c$ because the flow of returns on vacancy are perpetually the cost of maintaining it
- When $E \to 0$, $a = 0$ and $\alpha \to \infty$, so big fraction approaches one and $rV_V = y - (b + c)$



- Curve of rV_V as a function of E has shape shown above.

- Equilibrium occurs where value of additional vacancies is exactly zero, at E^*
- Effects of changes in parameters:

  $y\uparrow \Rightarrow rV_V$ curve shifts $\uparrow$

  $k\uparrow \Rightarrow rV_V$ curve shifts $\nearrow$

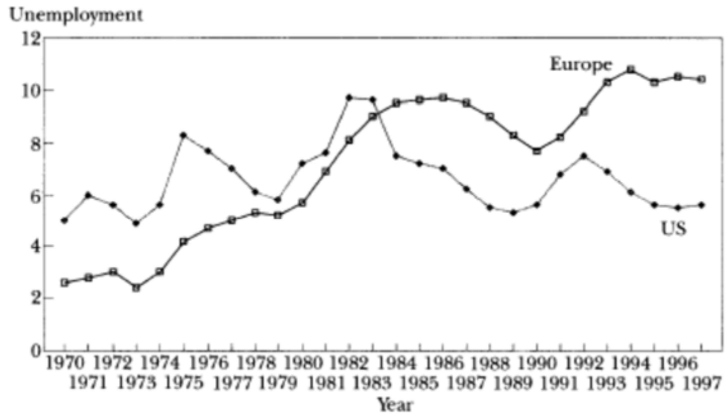
  $b\uparrow \Rightarrow rV_V$ curve shifts $\downarrow$

Applications

- Sectoral shifts
  - When the economy is undergoing a lot of structural shifts from one industry/region to another, k may fall as matching becomes harder
  - This would raise equilibrium unemployment in the model
- Active labor-market policies
  - Scandinavian countries have had good success with policies to facilitate job matching
  - This would be an increase in efficiency of matching so k increases
  - (U.S. effectiveness not so good)

Natural Unemployment: Empirical Evidence

- Based on Nickel and Siebert's papers in 1997 *JEP*
- Economists have been studying the high natural unemployment rate in Europe intensively since about 1990



- There is no single, simple explanation
  - For example, Spain and Portugal have quite similar institutions, but Spanish unemployment is twice as high
  - European institutions were similar in 1960s when unemployment was very low
- Candidates that are usually discussed
  - Employment protection
    - Firms that can't fire won't hire
  - Collective bargaining coverage
  - Generous unemployment benefits
  - Tax wedge
  - Lack of wage flexibility
    - Is European unemployment the mirror image of US wage stagnation?
    - In U.S., low-skill wages have fallen; in Europe, low-skill employment has stagnated
  - General lack of "flexible labor market"
    - Low churn
    - Low mobility
- Exceptions to the rule
  - Netherlands undertook flexible labor-market reforms that dropped unemployment a lot
  - Germany is now doing better, although absorption of East increased natural rate
  - Sweden has used active labor-market policies effectively