Ph.D. Macroeconomics — Fall 2015

Homework Assignment #1 — Due: 9:00, Thursday 8 October

1. **Marginal products in terms of the intensive production function.** In the Solow growth model, we write the aggregate production function as $Y = F(K, AL)$, and the intensive production function as $y = f(k)$, where $y \equiv \frac{Y}{AL}$, $k \equiv \frac{K}{AL}$, and $f(\bullet) \equiv F(\bullet, 1)$. Assume constant returns to scale and the usual properties for the partial derivatives of *F*.

a. Show that the marginal products of capital and labor are $\frac{\partial Y}{\partial K} = f'(k)$ and $\frac{\partial Y}{\partial L} = A\left[f(k) - kf'(k)\right]$.

b. Show that if, as we assume, $f'(k) > 0$ and $f''(k) < 0$, then an increase in *k* decreases the marginal product of capital and increases the marginal product of labor. What do these results mean in economic terms? Explain why they are intuitively reasonable.

c. Is *k* constant, increasing, or decreasing over time on a steady-state growth path? Is *A* constant, increasing, or decreasing over time on a steady-state growth path?

d. Two of Nicholas Kaldor's six famous "stylized facts" about modern growth are that the rate of return to a unit of capital is stable in the long run and that the real wage earned by a unit of labor (not effective labor) grows over time. In competitive equilibrium, each unit of capital and labor will be paid its marginal product, so the real wage w = MPL and the real rental rate on capital $r + \delta$ = MPK. Based on your results above, discuss whether or not the Solow model is consistent with these two stylized facts.

2. **Effects of changes in the Solow model.** Explain how, if at all, each of the following changes affects the breakeven and actual investment lines in the basic diagram for the Solow model. Then describe how, if at all, *k* would change both immediately and after the economy has returned to a steady-state path.

a. The rate of depreciation falls.

b. The rate of technological progress rises.

c. Workers exert more effort, so that more output is produced per unit of effective labor for any given amount of capital per unit of effective labor.

3. **Immigration.** Consider an economy with technological progress ($g > 0$) but with no ongoing population growth ($n = 0$) that is initially on its steady-state balanced growth path. Now suppose that immigration increases the population all at once from L_0 to L_1. The immigrants bring no capital with them and do not increase either the level or rate of growth of technological progress.

a. At the moment of the population increases, what happens to the level of *k* and the level of *y*? Why?

b. At the moment of the increase, what happens to *Y*? Why?

c. After the initial jump (if any) in *k*, how do *k* and *y* change over time? Why?

d. Once the economy has again reached a steady-state growth path, how do the levels of *k* and *y* compare with those before the immigration occurred? Why?

e. Compare and show on a graph (1) the new steady-state path for $\ln Y(t)$ and (2) the path $\ln Y(t)$ would have followed had there been no immigration. Be as specific as you can about any differences.

4. **Exploring Romer's growth model with finite natural resources.** In the final section of Chapter 1, Romer develops a version of the Solow model with land and natural resource inputs in the production function. With T denoting land input and R the flow of natural resources used up in production, he writes the production function as

$$Y(t) = K(t)^{\alpha} R(t)^{\beta} T(t)^{\gamma} \left[A(t)L(t)\right]^{1-\alpha-\beta-\gamma},$$

with $\alpha > 0$, $\beta > 0$, $\gamma > 0$, and $\alpha + \beta + \gamma < 1$. Romer assumes that $\dot{T}(t) = 0$ and that $\dot{R}(t) = -b < 0$ in the steady state, with L and A growing, as before, at n and g, respectively. The production function is assumed to follow the Cobb-Douglas form shown. As before, $\dot{K}(t) = sY(t) - \delta K(t)$.

a. If the world's stock of natural resources is finite, why must the growth of R in the steady state be negative? (Think about what happens as $t \to \infty$ if R is constant or growing.)
b. In the simple Solow model (without land or resources), both factors of production, capital K and effective labor AL grow at the same rate in the steady state, with output also growing at that rate. Is it possible to have a steady-state equilibrium in the resource model with all four factors (and output) growing at the same rate? Explain.
c. To describe the steady-state growth path in this model, we find conditions under which the growth rate of the capital stock $\dot{K}/K$ is constant. Use the equation for $\dot{K}(t)$ above to show that $\dot{K}(t)/K(t)$ is constant if and only if $Y(t)/K(t)$ is constant.
d. If $Y(t)/K(t)$ is to be constant, what must be true of the growth rates of Y and K? Why?
e. Use this condition to derive an expression for g_Y^*, the steady-state rate of output growth—the rate of output growth at which $Y(t)/K(t)$ is constant and therefore $\dot{K}(t)/K(t)$ is also constant.
f. Show that per-capita output grows at $g_{Y/L}^* = g - \dfrac{\beta b + (\beta + \gamma)(g + n)}{1 - \alpha} < g$ in the steady state.
g. What happens to the "growth drag" term (the one with the minus sign in the above equation) if β and γ are large? What does this mean in economic terms? What happens if $\beta = \gamma = 0$? Explain.