Problem 4.8-modified (Engel)

Calculate the probability that a particle in a one-dimensional box of length \(a\) is found between 0 and 0.25\(a\), and the probability that it is found between 0.25\(a\) and 0.5\(a\).

**Part A.** \(n = 1\), \(\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)\)

**Part B.** \(n = 3\), \(\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{3\pi x}{a}\right)\)

**Part C (my add'n).** \(n = 10\), \(\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{10\pi x}{a}\right)\)

**Solution**

**Strategy.**
Probability for finding particle inside a portion of the box is obtained by evaluating the *normalization* integral over this portion of the box.

**Execution.**

\[
\begin{align*}
n &= \{1, 3, 10\} \\
dens &= \left(\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)\right)^2 \\
P_{\text{left}} &= \int_{0}^{0.25a} dens \, dx \\
P_{\text{ctr}} &= \int_{0.25a}^{0.5a} dens \, dx
\end{align*}
\]

\[
\{0.0908451, 0.303052, 0.25\}
\]

\[
\{0.409155, 0.196948, 0.25\}
\]

**Comment.** Classical physics predicts that the particle can be found with equal probability at all points in the box. Since the left quarter and center-left quarters of the box encompass equal distances (0.25 \(a\)), the probability of finding the particle in either zone should be the same (0.25 \(a/a = 0.25\)).
The ground state, $n = 1$, gives very different probabilities: 0.09 and 0.41 for the left and left-center regions, respectively.

These probabilities reverse in the second excited state, $n = 2$: 0.30 and 0.20 for the left and left-center regions. Note that both are converging on 0.25.

The probabilities in the ninth excited state, $n = 10$, match the classical values. This blending of quantum physics into classical physics at high $n$ is an example of Bohr's Correspondence Principle.