Problem 4.6 - modified (Engel)

Evaluate the normalization integral for the eigenfunctions of $\hat{H}$ for the particle in a box $\psi_n(x) = A \sin(n \pi x / a)$ using the trigonometric identity $\sin^2 y = (1 - \cos 2y) / 2$.

My modification: don't bother with the trig identity. However, try two integrations. One with $n = 3$, and one with $n = n$.

Solution

Strategy.
The normalization integral is defined as $\int \psi^* \psi \, dx$. If the function is normalized, this integral will equal one.

Execution. $\psi$ is real, so the normalization integral is

$$\int \psi^2 \, dx = \int A^2 \sin^2(n \pi x / a) \, dx$$

If $n = 3$

$$\int_0^a \sin^2(3 \pi x / a) \, dx$$

If $n = n$

$$\int_0^a \sin^2(n \pi x / a) \, dx$$

Simplify $[\%]$, $n \in \text{Integers}$

$$\frac{1}{4} a \left( 2 - \frac{\sin(2n \pi)}{n \pi} \right)$$

Both results agree (which is to be expected since the integral does not depend on the precise value of $n$).

The normalization integral simplifies to:

$$= \frac{a^2}{2}$$

and the function will be normalized if $A = \sqrt{\frac{2}{a}}$.

Comment. The normalization constant is the same for all of the eigenfunctions. It does not depend on $n$. 