Problem 2.23 (Engel)

Normalize the functions $\phi_n(\theta) = e^{in\theta}$ for the case where $n$ is an integer (and $\theta$ ranges from $0 \to 2\pi$)

**Strategy.** I need to scale the function by multiplying it by a constant $N$ so that:

$$\int_0^{2\pi} (N^* \phi_n^*(\theta))(N \phi_n(\theta)) \, d\theta = 1$$

**Execution.**

$$1 = \int_0^{2\pi} (N^* \phi_n^*(\theta))(N \phi_n(\theta)) \, d\theta$$

$$= N^* N \int_0^{2\pi} \phi_n^*(\theta) \phi_n(\theta) \, d\theta = N^* N \int_0^{2\pi} (e^{-in\theta})(e^{in\theta}) \, d\theta$$

$$= N^* N \int_0^{2\pi} d\theta = N^* N (2\pi)$$

Because $N^* N$ is real, it is possible to choose $N$ to be real.

$$N = \frac{1}{\sqrt{2\pi}}$$

$$\int_0^{2\pi} e^{-in\theta} e^{in\theta} \, d\theta$$

$$2\pi$$