## Exercise 8: An Introduction to Descriptive and <br> Nonparametric Statistics

Elizabeth M. Jakob and Marta J. Hersek

## Goals of this lab

1. To understand why statistics are used
2. To become familiar with descriptive statistics
3. To become familiar with nonparametric statistical tests, how to conduct them, and how to choose among them
4. To apply this knowledge to sample research questions

## Background

If we are very fortunate, our experiments yield perfect data: all the animals in one treatment group behave one way, and all the animals in another treatment group behave another way. Usually our results are not so clear. In addition, even if we do get results that seem definitive, there is always a possibility that they are a result of chance.

Statistics enable us to objectively evaluate our results. Descriptive statistics are useful for exploring, summarizing, and presenting data. Inferential statistics are used for interpreting data and drawing conclusions about our hypotheses.

Descriptive statistics include the mean (average of all of the observations; see Table 8.1), mode (most frequent data class), and median (middle value in an ordered set of data). The variance, standard deviation, and standard error are measures of deviation from the mean (see Table 8.1). These statistics can be used to explore your data before going on to inferential statistics, when appropriate.

In hypothesis testing, a variety of statistical tests can be used to determine if the data best fit our null hypothesis (a statement of no difference) or an alternative hypothesis. More specifically, we attempt to reject one of these hypotheses. We calculate the test statistic appropriate for our research methods and the design of our study, and calculate the probability that the pattern we see in our data is due to chance alone. This probability is called the $P$ value. By convention, most behavioral ecologists agree that when $P$ is equal to or less than 0.05 , we can confidently reject the null hypothesis.

To determine which type of statistical test to use on a given set of data, we must first determine whether or not the data fit a normal (bell-shaped) distribution. If so, we can use parametric statistical tests (see Figure 8.1). If the data are not normal, we must use nonparametric tests. Since many of the data collected in animal behavior studies are not normally distributed, we will focus on nonparametric tests in this lab.

A flow chart to help you decide which tests to use is given in Figure 8.1. Following this is a series of worked examples for a number of nonparametric tests. Begin by acquainting yourself with the flow chart; then skip ahead to the Methods section that follows the worked examples.

## Here are some helpful terms:

Continuous data: numerical data, such as number of seconds, distance, or frequency of a behavior.
Categorical data: data that can be put into categories, such as number of animals that moved toward a stimulus, moved away from a stimulus, or stayed in place.
Ordinal data: categorical data where there is a logical ordering to the categories. A good example is the Likert scale that you see on many surveys: $1=$ Strongly disagree; $2=$ Disagree; $3=$ Neutral; $4=$ Agree; 5=Strongly agree.
Unpaired data: data points that are independent from each other, such as data generated by testing two separate groups of animals.
Paired data: data points that are naturally paired in some way, most commonly because the same animal was tested more than once. These data points should not be treated as independent from one another.
Number of groups: the number of different test groups being compared.


Figure 8.1. A flow chart to aid in deciding which statistical test is appropriate. Only common tests are included.

BEFORE CLASS: After examining the flow chart, look through the following tests.

## 1. Mann-Whitney $U$ test

This test is used to determine the significance of differences between two sets of unpaired data. A ranking system is used.

Example: You are interested in whether the movement rate of the protozoan Paramecium caudatum is influenced by whether they are tested under dim or bright light. The null hypothesis is $P$. caudatum has the same rate of movement under both conditions. You measure movement rate by counting the number of squares in a counting chamber a Paramecium crosses every 10 seconds.

1. First, order each group from smallest to largest. Next, rank the data of the two groups combined. The lowest score (of both groups) gets a value of 1 , the next highest (of both groups) a value of 2 , etc. In the case of ties (for example, two values of 12), each value is ranked, the ranks are averaged, and the average rank is assigned to each of the tied scores: $(11+12) / 2=11.5$. If you've done this properly, your last rank will equal $N$, the total number of samples.

## Example: P. caudatum movement data (squares crossed per 10 sec.)

| Dim Light | Rank | Bright Light | Rank |
| :---: | :---: | :---: | :---: |
| 10 | 7 | 5 | 1 |
| 11 | 9.5 | 6 | 2 |
| 12 | 11.5 | 7 | 3 |
| 12 | 11.5 | 8 | 4 |
| 15 | 13 | 9 | 5 |
| 16 | 14 | 10 | 7 |
| 17 | 15 | 10 | 7 |
|  |  | 11 | 9.5 |

2. Designate the sample size of the larger group as $N_{L}$ and that of the smaller as $N_{S}$. In our example $N_{L}=8$ and $N_{S}=7$.
3. Sum the ranks ( $T$ ) of each group.

$$
\begin{aligned}
& T_{S}=7+9.5+11.5+11.5+13+14+15=81.5 \\
& T_{L}=1+2+3+4+5+7+7+9.5=38.5
\end{aligned}
$$

4. Calculate the test statistics, $U_{S}$ and $U_{L}$.

$$
\begin{aligned}
& U_{S}=N_{S} N_{L}+\frac{N_{S}\left(N_{S}+1\right)}{2}-T_{S}=2.5 \\
& U_{L}=N_{S} N_{L}-U_{S}=53.5
\end{aligned}
$$

5. Choose the greater of the two values of $U$. This is the test statistic. Compare it to the critical value in Table 8.2. The test statistic must be higher than the critical value to be significant. In this example, the higher $U$ is 53.5. Look in Table 8.2 under $N_{L}=8$ and $N_{S}$ $=7$ at the $95 \%$ level $(P=0.05)$. The critical value for $P=0.05$ is 43 ; since $53.5>43$, we can reject the null hypothesis with $95 \%$ probability that rejection is correct. We conclude that Paramecium swim more slowly under bright light.

## 2. Kruskal-Wallis test

The Kruskal Wallis Test is similar to the Mann-Whitney U test, but here we have more than two groups. Work through the Mann-Whitney U example before attempting this one.

Example: You are interested in the antipredator behavior of garter snakes. You wonder how close you, as a simulated predator, can get before the snake crawls away. Because snakes are poikilotherms and can move more quickly when it is warmer, you suspect that this behavior is influenced by temperature. You compare three groups: snakes at $23^{\circ} \mathrm{C}, 25^{\circ} \mathrm{C}$, and $27^{\circ} \mathrm{C}$. The data are closest approach distance, in meters. The null hypothesis is that snakes tested under these three temperatures do not differ in how close an experimenter approaches before they flee.

1. First order and rank the data, as described for the Mann-Whitney $U$ test. When there are tied scores, each score is given the mean of the ranks for which it is tied. Compute the sum of the ranks for each group, symbolized by $R . R_{l}$ is the sum of the ranks of group 1 , etc.

## Example: Flight distance of snakes (in meters)

| $\mathbf{2 3}^{\circ} \mathbf{C}$ | $\boldsymbol{\operatorname { a n } \boldsymbol { k }}$ | $\mathbf{2 5}^{\circ} \mathbf{C}$ | $\boldsymbol{\operatorname { R a n k }}$ | $\mathbf{2 7}^{\circ} \mathbf{C}$ | Rank |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | 1 | 0.75 | 2 | 3.5 | 7 |
| 1 | 3 | 3.25 | 6 | 5.5 | 12 |
| 1.25 | 4 | 4 | 8 | 6 | 13 |
| 3 | 5 | 4.75 | 10 | 8 | 14 |
| 4.25 | 9 | 5.25 | 11 |  |  |
| $=46$ |  |  |  |  |  |

2. Now compute the test statistic, H , using the following formula:

$$
H=\frac{12}{N(N+1)} \sum \frac{R_{i}^{2}}{n_{i}}-3(N+1)
$$

In this formula, the $\sum$ is a summation sign, and indicates that you should sum up each $R^{2}$ value, from $R_{l}$ to $R_{3}$. Plugging in the appropriate numbers for $R, N$ (the total number of observations), and $n_{i}$ (the number of observations in each group):

$$
H=\frac{12}{14(14+1)}\left[\frac{(22)^{2}}{5}+\frac{(37)^{2}}{5}+\frac{(46)^{2}}{4}\right]-3(14+1)=6.4
$$

If you have a large number of ties, use the correction for ties. Compute $H$ as above, then divide by

$$
1-\frac{\sum\left(t^{3}-t\right)}{N^{3}-N}
$$

where $t=$ the number of observations in a tied group of scores
$N=$ the total number of all observations
3. Compare your test statistic with Table 8.3. The test statistic must be higher than the critical value to be significant. $H$, at 6.4 , is greater than 5.6429 , so you may reject the null hypothesis at $P<.05$. The three groups do differ.

## 3. Sign test

The sign test is used for two-groups when the data are paired. In this test, only the signs of the differences are used. Another nonparametric test, the Wilcoxon matched-pairs signed rank test, is more powerful because it uses both the signs and the magnitude of the differences. We will use the sign test as a general example of how paired data can be treated.

Example: You imagine that male mice might benefit from avoiding inbreeding, or mating with close relatives. Because mice depend on odor for a great deal of their information about the world, you decide to present males with soiled litter from the cages of females. You test each male twice: once with litter from his sister, and once with litter from a stranger. The females are sexually receptive, so the soiled litter should be rich in chemical cues. You present the litter in random order so that half the males get their sibling's litter first, and half get the stranger's litter first. Since the same males are tested twice, a Mann-Whitney U test is inappropriate. Null hypothesis: The number of sniffs per minute will be the same when males are exposed to the litter of their sisters vs. that of strangers.

| Male ID <br> Number | Number of Sniffs/Min <br> with Sister's Litter | Number of Sniffs/Min <br> with Stranger's Litter | Sign of the <br> Difference |
| :---: | :---: | :---: | :---: |
| 1 | 10 | 9 | + |
| 2 | 8 | 3 | + |
| 3 | 3 | 5 | - |
| 4 | 20 | 11 | + |
| 5 | 15 | 9 | + |
| 6 | 35 | 21 | + |
| 7 | 4 | 6 | - |
| 8 | 11 | 10 | + |
| 9 | 41 | 20 | + |
| 10 | 22 | 21 | + |
| 11 | 16 | 16 | 0 |
| 12 | 18 | 17 | + |
| 13 | 7 | 0 | + |
| 14 | 11 | 5 | + |

1. Subtract one data column from the other to determine the sign of the difference. (It doesn't matter which you subtract from which, just be consistent.)
2. Note the least frequent sign. In this case, the least frequent sign is negative, and there are two. The test statistic, $x$, therefore equals 2 .
3. Determine N , the number of pairs which showed a difference. Here we disregard male \#11, so $N=13$.
4. Look at Table 8.4 for $N=13$ along the left-hand side. Now find $x=2$. The $P$ value is 0.011 (the initial decimal places are omitted in the table to save space). You can therefore reject your null hypothesis at the 0.05 level.

## 4. Chi-square test of independence and chi-square goodness-of-fit test

Tests using the chi-square statistic are useful when you have nominal data (categories rather than numbers). For example, a category might be "large" vs. "small," "laid eggs" vs. "did not lay eggs," etc.

First we will look at the chi-square test of independence. This test helps us determine whether two variables are associated. If two variables are not associated (that is, they are independent), knowing the value of one variable will not help us determine the value of the other variable.

Example: When snails sense the presence of a nearby starfish, a predator, via chemicals in the water, they will climb. We can look at three groups of snails: the first group is the control group, with the snails exposed to plain sea water, the second group is exposed to water scented by a sea urchin (an herbivore), and the third group to sea water scented by a predatory starfish. The data collected for each snail is whether it climbed or not. These are categorical data: the snail could do one thing or the other. The categories are mutually exclusive (the snail could not "climb" and "not climb"). If the variables are independent, there will be no relationship between the type of water the snail is exposed to (the first variable) and how it responds (the second variable). Note: if instead of making categories of "climb" and "not climb," the experimenter had measured the distance each snail moved, the chi-square test would be inappropriate. (Which test should be used for those data?)

1. Make a table of observed frequencies, the data actually collected in the experiment.

Observed frequencies

|  | Source of Test Water |  |  | Row totals |
| :--- | :---: | :---: | :---: | :---: |
| Behavior | Control | Sea urchin | Predator |  |
| Climb | 12 | 14 | 24 | 50 |
| Not climb | 28 | 23 | 15 | 66 |
| Column totals | 40 | 37 | 39 | Grand total $=116$ |

Note: The grand total of the rows should equal the grand total of the columns.
2. Calculate and tabulate the expected frequency for each category (for the number of snails observed, the frequency expected in each category if there is no relationship between the variables):

```
column total x row total
    grand total
```


## Expected frequencies

| Behavior | Source of Test Water |  |  | Row totals |
| :--- | :---: | :---: | :---: | :---: |
|  | Control | Sea urchin | Predator |  |
|  | 17.2 | 16 | 16.8 | 50 |
| Not climb | 22.8 | 21 | 22.2 | 66 |
| Column totals | 40 | 37 | 39 | Grand total $=116$ |

3. Calculate the value of chi-square $\left(x^{2}\right)$ :

$$
\begin{aligned}
& \quad \chi^{2}=\sum \frac{(O-E)^{2}}{E} \\
& \text { where: } \\
& O=\text { the observed frequency in each cell } \\
& E=\text { the expected frequency in each cell } \\
& \chi^{2}=\frac{(12-17.2)^{2}}{17.2}+\frac{(28-22.8)^{2}}{22.8}+\frac{(14-16)^{2}}{16}+\frac{(23-21)^{2}}{21}+\frac{(24-16.8)^{2}}{16.8}+\frac{(15-22.2)^{2}}{22.2} \\
& x^{2}=8.62
\end{aligned}
$$

4. Examine the table of critical values for this test (see Table 8.5). The $d f$ column corresponds to the degrees of freedom for this test. Degrees of freedom is a number that results from the way the data are organized, and refers to whether the observations are free to vary. For example, if all of 50 observations must fall into two categories, as soon as we know that one category holds 41 data points, then the other category holds nine. For every statistical test, there are established methods for determining degrees of freedom. For the chi-square test, the formula is:

$$
d f=(\# \text { rows }-1)(\# \text { columns }-1)=(2-1)(3-1)=2
$$

We compare the test statistic to the critical value: if it is bigger, we reject the null hypothesis. The calculated $x^{2}$ is 8.62 , which is greater than 5.99 . The three groups of snails moved differently.

A second type of chi-square test is called the chi-square goodness-of-fit test. In this case, the experimenter tests to see how the data match expected values that were determined before the test was run. For example, in Mendelian genetics, we can predict the outcome of different crosses; the ratio of the different types of offspring is known in advance. In this case, we compare the observed values from the experiment with the expected values, generated by theory. The calculations are performed in exactly the same way as for the chi-square test of independence.

## 5. The binomial test

This test is useful for categorical data where we have only two categories, and when we are interested in testing whether the data are equally likely to fall into either category.

Example: You've been using a coin to randomly assign treatments to your experimental animals, but you are beginning to suspect that the coin is not fair, and you decide you'd better test this. The null hypothesis is: the coin is equally likely to come up tails as heads.

1. Flip the coin 11 times. Nine times it comes up heads, and twice it comes up tails.
2. Using Table 8.4 , locate the value for $N$ (in this case, 11) along the left side, and the smallest numerical score ( $x$; in this case, 2) along the top. The probability associated with this distribution is 0.033 (i.e., $P=0.033$ ). Because $P<0.05$, we can reject our null hypothesis: the coin is not fair.

## IN CLASS

After you have reviewed the flow chart and glanced through the worked examples, attempt the following problems. In each, an experiment is described. Determine which statistical test is most appropriate, and answer all questions posed. Refer back to the worked examples to help you understand how to conduct each test.

1. Elephants make low-frequency sounds, inaudible to humans. Apparently these sounds are used in long-distance communication between individuals. You are interested in the response of bull and female elephants to the sound of a female who is ready to mate. You mount a giant speaker on top of your van and drive around the plains looking for elephants. When you find one, you stop 15 m away, play the sound and watch the elephant's response. You discover:

9 bull elephants approach the van
2 bull elephants do not approach the van
3 female elephants approach the van
11 female elephants do not approach the van
Your experiment ends prematurely when one of the bull elephants, apparently enraged by the absence of a female, tips the van over and damages the speaker. You hope that you have enough data to make a claim about males and females.
a. What is the null hypothesis?
b. What statistical test should you use?
c. Calculate your test statistic. Is your result statistically significant?
d. What conclusion can you draw from this experiment?
2. Male butterflies sometimes court females of other species with similar wing patterns. You are interested in how long males persist in courting the wrong female. You decide to test each male with a dead female, to control for the effect of the female's behavior. You use three types of test females: one from the same species as the males, one from a different species with a similar wing pattern, and one from a different species with a different wing pattern. Each pair is placed in a cage, and you measure courtship time in seconds.

Female of same species: $23,20,17,25,28$
Female of different species, similar pattern: 18, 27, 24, 21
Female of different species, different pattern: 22, 21, 23, 20
a. Calculate the mean, variance, and standard deviation for each group.
b. Qualitatively compare the means and standard deviations for each group. (Do they look very different? Very similar?)
c. Which statistical test would you use to look for differences?
d. Perform the test. What is your test statistic? Can you reject your null hypothesis?
e. Give a biological reason why your test may have come out the way it did.
3. Honeybees returning from foraging convey information to bees in the hive about the location of food resources. One way they do this is through a waggle dance that other bees watch. Another way they convey information is by regurgitating some of the food they have collected to other bees, a process known as trophyllaxis. You are interested in the speed at which bees find a resource another bee "tells" them about. You decide to compare bees that have only observed a dance with bees that have observed a dance and accepted regurgitated food. You mark a lot of bees with bee tags (little numbered discs that you glue to the back of the thorax). This enables you to watch the same individual repeatedly. One day you choose a lot of bees that have seen a waggle dance but not accepted food. You measure (in seconds) how long it takes for them to find the resource. A week later you go back to the hive, and find the same individuals. This time you watch until they see a dance and accept food, and again measure how long it takes them to reach the resource.

Here are your data. The numbers are seconds needed for the bee to reach the resource.

| Bee \# | Watch Only | Watch and Accept Food |
| :---: | :---: | :---: |
| 1 | 87 | 80 |
| 2 | 53 | 48 |
| 3 | 57 | 57 |
| 4 | 89 | 88 |
| 5 | 48 | 38 |
| 6 | 109 | 160 |
| 7 | 109 | 100 |
| 8 | 48 | 78 |
| 9 | 29 | 26 |
| 10 | 45 | 41 |
| 11 | 67 | 53 |
| 12 | 120 | 98 |
| 13 | 55 | 55 |
| 14 | 89 | 78 |

a. What sort of data are these? Which test should you choose?
b. What is the test statistic? The table statistic?
c. You decide that a bee that has both watched and gotten food from another bee finds the resource faster than one that has just watched. What other factor that is a result of your experimental protocol might also explain your results?

Table 8.1 Formulas for descriptive statistics
$Y_{i}$ is an observation, or data point. The first observation is $Y_{1}$, the second is $Y_{2}$, etc.
$N$ is the sample size, or the number of observations.

## Mean:

$$
\bar{Y}=\frac{\sum Y_{i}}{N}
$$

Variance:

$$
s^{2}=\frac{\sum\left(Y_{i}-\bar{Y}\right)^{2}}{N-1}
$$

## Standard deviation:

$$
s=\sqrt{s^{2}}
$$

## Standard error:

$$
\frac{s}{\sqrt{N}}
$$

Median: Rank the values from lowest to highest and take the center-most value.

Mode: The most common value.

Table 8.2 Critical values of U, the Mann-Whitney statistic for $P=0.05$ and 0.01 . (Modified from Table 29, F.J. Rohlf and R.R. Sokal. 1981. Statistical Tables, 2nd edition. W.H. Freeman and Company.)

| $\mathbf{N}_{L}$ | $\mathbf{N}_{S}$ | $P=0.05$ | $P=0.01$ |
| :---: | :---: | :---: | :---: |
| 3 | 2 |  |  |
|  | 3 | 9 |  |
| 4 | 2 |  |  |
|  | 3 | 12 |  |
|  | 4 | 15 |  |
| 5 | 2 | 10 |  |
|  | 3 | 14 |  |
|  | 4 | 18 | 20 |
|  | 5 | 21 | 24 |
| 6 | 2 | 12 |  |
|  | 3 | 16 |  |
|  | 4 | 21 | 23 |
|  | 5 | 25 | 28 |
|  | 6 | 29 | 33 |
| 7 | 2 | 14 |  |
|  | 3 | 19 | 21 |
|  | 4 | 24 | 27 |
|  | 5 | 29 | 32 |
|  | 6 | 34 | 38 |
|  | 7 | 38 | 43 |
| 8 | 2 | 15 |  |
|  | 3 | 21 | 24 |
|  | 4 | 27 | 30 |
|  | 5 | 32 | 36 |
|  | 6 | 38 | 42 |
|  | 7 | 43 | 49 |
|  | 8 | 49 | 55 |
| 9 | 2 | 17 |  |
|  | 3 | 23 | 26 |
|  | 4 | 30 | 33 |
|  | 5 | 36 | 40 |
|  | 6 | 42 | 47 |
|  | 7 | 48 | 54 |
|  | 8 | 54 | 61 |
|  | 9 | 60 | 67 |
| 10 | 2 | 19 |  |
|  | 3 | 26 | 29 |
|  | 4 | 33 | 37 |
|  | 5 | 39 | 44 |
|  | 6 | 46 | 52 |
|  | 7 | 53 | 59 |
|  | 8 | 60 | 67 |
|  | 9 | 66 | 74 |
|  | 10 | 73 | 81 |


| $\mathbf{N}_{L}$ | $\mathbf{N}_{S}$ | 0.05 | 0.01 |
| :---: | :---: | :---: | :---: |
| 11 | 2 | 21 |  |
|  | 3 | 28 | 32 |
|  | 4 | 36 | 40 |
|  | 5 | 43 | 48 |
|  | 6 | 50 | 57 |
|  | 7 | 58 | 65 |
|  | 8 | 65 | 73 |
|  | 9 | 72 | 81 |
|  | 10 | 79 | 88 |
|  | 11 | 87 | 96 |
| 12 | 2 | 22 |  |
|  | 3 | 31 | 34 |
|  | 4 | 39 | 42 |
|  | 5 | 47 | 52 |
|  | 6 | 55 | 61 |
|  | 7 | 63 | 70 |
|  | 8 | 70 | 79 |
|  | 9 | 78 | 87 |
|  | 10 | 86 | 96 |
|  | 11 | 94 | 104 |
|  | 12 | 102 | 113 |
| 13 | 2 | 24 | 26 |
|  | 3 | 33 | 37 |
|  | 4 | 42 | 47 |
|  | 5 | 50 | 56 |
|  | 6 | 59 | 66 |
|  | 7 | 67 | 75 |
|  | 8 | 76 | 84 |
|  | 9 | 84 | 94 |
|  | 10 | 93 | 103 |
|  | 11 | 101 | 112 |
|  | 12 | 109 | 121 |
|  | 13 | 118 | 130 |
| 14 | 2 | 25 | 28 |
|  | 3 | 35 | 40 |
|  | 4 | 45 | 50 |
|  | 5 | 54 | 60 |
|  | 6 | 63 | 71 |
|  | 7 | 72 | 81 |
|  | 8 | 81 | 90 |
|  | 9 | 90 | 100 |
|  | 10 | 99 | 110 |
|  | 11 | 108 | 120 |
|  | 12 | 117 | 130 |
|  | 13 | 126 | 139 |
|  | 14 | 135 | 149 |

Table 8.3 Probabilities associated with values as large as observed values of H in Kruskal-Wallis tests.
(Modified from Table O in S. Siegel. 1956. Nonparametric Statistics for the Behavioral Sciences. McGraw-Hill, New York.)

| Sample Sizes |  |  | H | $\boldsymbol{P}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{N}_{1}$ | $\mathrm{N}_{2}$ | $\mathbf{N}_{3}$ |  |  |
| 2 | 1 | 1 | 2.7000 | . 500 |
| 2 | 2 | 1 | 3.6000 | . 200 |
| 2 | 2 | 2 | 4.5714 | . 067 |
|  |  |  | 3.7143 | . 200 |
| 3 | 1 | 1 | 3.2000 | . 300 |
| 3 | 2 | 1 | 4.2857 | . 100 |
|  |  |  | 3.8571 | . 133 |
| 3 | 2 | 2 | 5.3572 | . 029 |
|  |  |  | 4.7143 | . 048 |
|  |  |  | 4.5000 | . 067 |
|  |  |  | 4.4643 | . 105 |
| 3 | 3 | 1 | 5.1429 | . 043 |
|  |  |  | 4.5714 | . 100 |
|  |  |  | 4.0000 | . 129 |
| 3 | 3 | 2 | 6.2500 | . 011 |
|  |  |  | 5.3611 | . 032 |
|  |  |  | 5.1389 | . 061 |
|  |  |  | 4.5556 | . 100 |
|  |  |  | 4.2500 | . 121 |
| 3 | 3 | 3 | 7.2000 | . 004 |
|  |  |  | 6.4889 | . 011 |
|  |  |  | 5.6889 | . 029 |
|  |  |  | 5.6000 | . 050 |
|  |  |  | 5.0667 | . 086 |
|  |  |  | 4.6222 | . 100 |
| 4 | 1 | 1 | 3.5714 | . 200 |
| 4 | 2 | 1 | 4.8214 | . 057 |
|  |  |  | 4.5000 | . 076 |
|  |  |  | 4.0179 | . 114 |
| 4 | 2 | 2 | 6.0000 | . 014 |
|  |  |  | 5.3333 | . 033 |
|  |  |  | 5.1250 | . 052 |
|  |  |  | 4.4583 | . 100 |
|  |  |  | 4.1667 | . 105 |

Sample Sizes

| 崖 |  |  | H | P |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}_{1}$ | $\mathrm{N}_{2}$ | $\mathbf{N}_{3}$ |  |  |
| 4 | 3 | 1 | 5.8333 | . 021 |
|  |  |  | 5.2083 | . 050 |
|  |  |  | 5.0000 | . 057 |
|  |  |  | 4.0556 | . 093 |
|  |  |  | 3.8889 | . 129 |
| 4 | 3 | 2 | 6.4444 | . 008 |
|  |  |  | 6.3000 | . 011 |
|  |  |  | 5.4444 | . 046 |
|  |  |  | 5.4000 | . 051 |
|  |  |  | 4.5111 | . 098 |
|  |  |  | 4.4444 | . 102 |
| 4 | 3 | 3 | 6.7455 | . 010 |
|  |  |  | 6.7091 | . 013 |
|  |  |  | 5.7909 | . 046 |
|  |  |  | 5.7273 | . 050 |
|  |  |  | 4.7091 | . 092 |
|  |  |  | 4.7000 | . 101 |
| 4 | 4 | 1 | 6.6667 | . 010 |
|  |  |  | 6.1667 | . 022 |
|  |  |  | 4.9667 | . 048 |
|  |  |  | 4.8667 | . 054 |
|  |  |  | 4.0667 | . 102 |
| 4 | 4 | 2 | 7.0364 | . 006 |
|  |  |  | 6.8727 | . 011 |
|  |  |  | 5.4545 | . 046 |
|  |  |  | 5.2364 | . 052 |
|  |  |  | 4.5545 | . 098 |
|  |  |  | 4.4455 | . 103 |
| 4 | 4 | 3 | 7.1439 | . 010 |
|  |  |  | 7.1364 | . 011 |
|  |  |  | 5.5985 | . 049 |
|  |  |  | 5.5758 | . 051 |
|  |  |  | 4.5455 | . 099 |
|  |  |  | 4.4773 | . 102 |
| 4 | 4 | 4 | 7.6539 | . 008 |
|  |  |  | 7.5385 | . 011 |
|  |  |  | 5.6923 | . 049 |
|  |  |  | 5.6538 | . 054 |
|  |  |  | 4.6539 | . 097 |
|  |  |  | 4.5001 | . 104 |
| 5 | 1 | 1 | 3.8571 | . 143 |

Sample Sizes

| Sizes |  |  | H | P |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}_{1}$ | $\mathrm{N}_{2}$ | $\mathbf{N}_{3}$ |  |  |
| 5 | 2 | 1 | 5.2500 | . 036 |
|  |  |  | 5.0000 | . 048 |
|  |  |  | 4.4500 | . 071 |
|  |  |  | 4.2000 | . 095 |
|  |  |  | 4.0500 | . 119 |
| 5 | 2 | 2 | 6.5333 | . 008 |
|  |  |  | 6.1333 | . 013 |
|  |  |  | 5.1600 | . 034 |
|  |  |  | 5.0400 | . 056 |
|  |  |  | 4.3733 | . 090 |
|  |  |  | 4.2933 | . 122 |
| 5 | 3 | 1 | 6.4000 | . 012 |
|  |  |  | 4.9600 | . 048 |
|  |  |  | 4.8711 | . 052 |
|  |  |  | 4.0178 | . 095 |
|  |  |  | 3.8400 | . 123 |
| 5 | 3 | 2 | 6.9091 | . 009 |
|  |  |  | 6.8218 | . 010 |
|  |  |  | 5.2509 | . 049 |
|  |  |  | 5.1055 | . 052 |
|  |  |  | 4.6509 | . 091 |
|  |  |  | 4.4945 | . 101 |
| 5 | 3 | 3 | 7.0788 | . 009 |
|  |  |  | 6.9818 | . 011 |
|  |  |  | 5.6485 | . 049 |
|  |  |  | 5.5152 | . 051 |
|  |  |  | 4.5333 | . 097 |
|  |  |  | 4.4121 | . 109 |
| 5 | 4 | 1 | 6.9545 | . 008 |
|  |  |  | 6.8400 | . 011 |
|  |  |  | 4.9855 | . 044 |
|  |  |  | 4.8600 | . 056 |
|  |  |  | 3.9873 | . 098 |
|  |  |  | 3.9600 | . 102 |
| 5 | 4 | 2 | 7.2045 | . 009 |
|  |  |  | 7.1182 | . 010 |
|  |  |  | 5.2727 | . 049 |
|  |  |  | 5.2682 | . 050 |
|  |  |  | 4.5409 | . 098 |
|  |  |  | 4.5182 | . 101 |

Sample Sizes

| $\mathbf{N}_{I}$ | $\mathbf{N}_{2}$ | $\mathbf{N}_{\mathbf{3}}$ | $\boldsymbol{H}$ | $\boldsymbol{P}$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 4 | 3 | 7.4449 | .010 |
|  |  |  | 7.3949 | .011 |
|  |  |  | 5.6564 | .049 |
|  |  |  | 5.6308 | .050 |
|  |  |  | 4.5487 | .099 |
|  |  |  | 4.5231 | .103 |
|  |  |  |  | 7.7604 |

Table 8.4 Table of probabilities associated with values as small as observed values of $x$, for use in sign test and binomial test. Values for total sample size are in the left-hand column, and values for $x$ are across the top. Decimal places are omitted in order to save space.
(Table D in S. Siegel. 1956. Nonparametric Statistics for the Behavioral Sciences. McGraw-Hill, New York.)

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 031 | 188 | 500 | 812 | 969 | $*$ |  |  |  |  |  |  |  |  |  |  |
| 6 | 016 | 109 | 344 | 656 | 891 | 984 | $*$ |  |  |  |  |  |  |  |  |  |
| 7 | 008 | 062 | 227 | 500 | 773 | 938 | 992 | $*$ |  |  |  |  |  |  |  |  |
| 8 | 004 | 035 | 145 | 363 | 637 | 855 | 965 | 996 | $*$ |  |  |  |  |  |  |  |
| 9 | 002 | 020 | 090 | 254 | 500 | 746 | 910 | 980 | 998 | $*$ |  |  |  |  |  |  |
| 10 | 001 | 011 | 055 | 172 | 377 | 623 | 828 | 945 | 989 | 999 | $*$ |  |  |  |  |  |
| 11 |  | 006 | 033 | 113 | 274 | 500 | 726 | 887 | 967 | 994 | $*$ | $*$ |  |  |  |  |
| 12 |  | 003 | 019 | 073 | 194 | 387 | 613 | 806 | 927 | 981 | 997 | $*$ | $*$ |  |  |  |
| 13 |  | 002 | 011 | 046 | 133 | 291 | 500 | 709 | 867 | 954 | 989 | 998 | $*$ | $*$ |  |  |
| 14 |  | 001 | 006 | 029 | 090 | 212 | 395 | 605 | 788 | 910 | 971 | 994 | 999 | $*$ | $*$ |  |
| 15 |  |  | 004 | 018 | 059 | 151 | 304 | 500 | 696 | 849 | 941 | 982 | 996 | $*$ | $*$ | $*$ |
| 16 |  |  | 002 | 011 | 038 | 105 | 227 | 402 | 598 | 773 | 895 | 962 | 989 | 998 | $*$ | $*$ |
| 17 |  |  | 001 | 006 | 025 | 072 | 166 | 315 | 500 | 685 | 834 | 928 | 975 | 994 | 999 | $*$ |
| 18 |  |  | 001 | 004 | 015 | 048 | 119 | 240 | 407 | 593 | 760 | 881 | 952 | 985 | 996 | 999 |
| 19 |  |  |  | 002 | 010 | 032 | 084 | 180 | 324 | 500 | 676 | 820 | 916 | 968 | 990 | 998 |
| 20 |  |  |  | 001 | 006 | 021 | 058 | 132 | 252 | 412 | 588 | 748 | 868 | 942 | 979 | 994 |
| 21 |  |  |  | 001 | 004 | 013 | 039 | 095 | 192 | 332 | 500 | 668 | 808 | 905 | 961 | 987 |
| 22 |  |  |  |  | 002 | 008 | 026 | 067 | 143 | 262 | 416 | 584 | 738 | 857 | 933 | 974 |
| 23 |  |  |  |  | 001 | 005 | 017 | 047 | 105 | 202 | 339 | 500 | 661 | 789 | 895 | 953 |
| 24 |  |  |  |  | 001 | 003 | 011 | 032 | 076 | 154 | 271 | 419 | 581 | 729 | 846 | 924 |
| 25 |  |  |  |  |  | 002 | 007 | 022 | 054 | 115 | 212 | 345 | 500 | 655 | 788 | 885 |

*1 or approximately 1 .

Table 8.5 Table of probabilities for the chi-square distribution.
(Modified from Table 14, Rohlf, F.J. and R.R. Sokal. 1981. Statistical Tables, 2nd edition. W. H. Freeman and Company.)

| Degrees of freedom | $\boldsymbol{P}=\mathbf{0 . 0 5}$ | $\boldsymbol{P}=\mathbf{0 . 0 1}$ |
| :---: | :---: | :---: |
| 1 | 3.841 | 6.635 |
| 2 | 5.991 | 9.210 |
| 3 | 7.815 | 11.345 |
| 4 | 9.488 | 13.277 |
| 5 | 11.070 | 15.086 |
| 6 | 12.592 | 16.812 |
| 7 | 14.067 | 18.475 |
| 8 | 15.507 | 20.090 |
| 9 | 16.919 | 21.666 |
| 10 | 18.307 | 23.209 |

