A Strategy for Mathematical Problem Solving – The First Step is Admitting You Have a Problem

1. **Read the problem carefully**. In particular, make sure that you understand what the problem is asking you to find. What would a reasonable answer look like? What units should your answer have?
2. **Assign variable names to relevant quantities**. This will certainly include the unknown value that you are asked to find, but may also include other related quantities. Be sure to clearly identify which quantity is represented by each variable.
3. **Write an equation relating these quantities**. This is often the most difficult step. Sometimes you can *translate* the sentences in the problem directly into an equation. Other times, you may have to use a math or science rule that isn't explicitly stated in the problem.
4. **Solve the equation for the desired quantity**. Usually, this means doing some algebra (multiplying both sides by the same number, etc.)
5. **Answer the question.** Remember what the problem is asking you to find. Sometimes, you may have to do an extra step to arrive at a final answer.
6. **Check your answer**. Does your answer seem like a reasonable response to the question you were originally asked? Are the units appropriate?

"Translating" between English and Math-ese

English is a nuanced language, and unfortunately there isn't an entirely automatic method for converting a word problem into an equation. However, certain keywords do provide valuable clues for arriving at an appropriate equation.

Operation:	Keywords:
Addition	Increased by More than Combined Together Total of Sum Added to
Subtraction	Decreased by Less than, fewer than Minus, less Difference
Multiplication	**Of** ("half of 50") Times, multiplied by Product of Increased/decreased by a **factor** of
Division	**Per** ("35 miles per gallon") a (as in "3 dollars a gallon")

	ratio of quotient of **percent** (read as "per cent", meaning divide by 100)
Equals	Is, are, was, were, will be Gives, yields
A variable (x, etc.)	What What number

Percent Problems

This technique of "translating" English sentences into math equations using keywords works especially well for applications involving percentages. Statements involving percentages can often be distilled down to a sentence like

12 is 60 percent of 20

Typically, percent problems involve a statement like the above, but with one of the three numbers unknown.

What is 60 percent of 20?
12 is what percent of 20?
12 is 60 percent of what number?

These are the three basic types of percentage questions. **Luckily, all can be solved using a single strategy: translate the question into an algebra equation and solve for the unknown variable.**

Example:	48	is	80	percent	of	what number?
Translate:	45	=	80	/100	*	x

If you are given a problem involving percents, first see if you can reword it to match one of the three forms given above. If so, the translate-and-solve strategy will work every time.

A Warning About Percents

When working problems involving percentages, a common mistake is to lose track of what the percent is referring to. (70 percent of *what*?)

Example: A textbook at the Reed bookstore is marked down by 20%. Its sale price is \$160. What was its original price?

A very common error is to find 20% of \$160 (which is \$32), and then add that to the sale price (giving an original price of \$160+\$32=\$192). But if we take 20% off \$192, we get \$153.60, which is not the given sale price. *What happened?*

Remember, a sale price is found by calculating 20% of the *original price* and subtracting it. It is not found by calculating 20% of the sale price. (After all, if we knew the sale price we wouldn't need to be doing more calculations to find it!)

A better way to approach this problem is to first ask yourself "20% of *what*?" Well, 20% of the original price, of course! We can now rephrase the problem:

The original price,	minus 20%	of	the original price,	is	$160.
x	- 20/100	*	x	=	160

Since the original price is our desired quantity, let's call it *x*. We can translate the above to

Percentage problems always include a part and a whole. (e.g., "the part is 50% of the whole.") Be very careful that you understand which quantity is the part and which is the whole in a given problem.

Direct and Inverse Variation

Two quantities are said to be *directly proportional* (or to *vary directly*) if a change in one produces a proportional change in the other. For example, the distance *d* of a car trip and the amount of time *t* for the trip may be directly proportional. Increasing the distance by a factor of 3 will probably increase the time by a factor of 3 as well. We can express this in an equation as

$$d=k*t$$

where k is some fixed number, which we call the *proportionality constant*. For this example, k happens to equal the average velocity of the car. If k=50 km/hr, then we have

$$d=50t$$

for any choice of d and t. If t doubles from 2 hours to 4 hours, this equation ensures that d will also double (from 100 km to 200 km).

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Two quantities are *inversely proportional* (or *vary inversely*) if an increase in one produces a proportional *decrease* in the other. In our driving example, velocity and time are inversely proportional: doubling the driving speed will cut the trip time in half. Similarly, doubling the trip time requires halving the driving speed.

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English wording	Suitable equations (k constant)
x and y are directly proportional	x/y=k x=k*y
x and y are inversely proportional	x*y=k x=k/y y=k/x

Solving Problems with Direct and Inverse Variation

When dealing with direct and inverse variation, we encounter two types of problems. In the first case, we are told that two quantities are directly or inversely proportional.

Example: When buying grapes at the supermarket, the weight of the grapes and the price of the grapes is directly proportional. Find an equation to describe this situation.
Solution: Let W=the weight of the grapes and P=the price. To be directly proportional, their *ratio* must remain constant. That is,

$$P/W=k$$

Where k is a constant. (Another possible equation is W/P=k, but in this case the constant k would have a different value.)
Check: Choose some values for P, W, and k. Does a change in P yield a proportional change in W? If so, then this equation does satisfy the definition of direct variation.
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Example: According to Boyle's Law, at constant temperature, the absolute pressure and the volume of a gas are inversely proportional. Find an equation to describe this law.
Solution: Let P=pressure and V=volume. In order for these quantities to be inversely proportional, their *product* must remain constant.

$$PV=k$$

where k is a constant (that depends on the specific situation.)
Check: Choose some values for P, V and k. Does an increase in P produce a proportional decrease in V?
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However, sometimes we have to first interpret the information in a problem to determine if a direct or inverse variation model is appropriate.

Example: It takes 4 workers 6 hours to repair a road. How long will it take 7 workers to do the job if they work at the same rate?
Solution: Let w=the number of workers and t=the time required to fix the road. We first need to decide if these quantities are directly proportional, inversely proportional, or neither. To answer this, ask "what effect would doubling the number of workers have on the time required to repair the road?" I would expect that increasing the number of workers should *decrease* the time for repairs, probably proportionally. So in this case, an inverse variation model is best.
Remember, in an inverse variation model, the *product* of the variables should remain constant. So

$$w*t=k$$

for a constant k. How do we find k? Since we know that 4 workers require 6 hours, we can substitute these figures for w and t, obtaining

$$4*6=k$$

So k=24. If we have 7 people working, then our model gives

$$7*t=24$$

Solving for t, we get a repair time of 24/7, or about 3.42 hours.